Week 6 Class Exercise

Total sleep time per night among college students was approximately Normally distributed with mean $\mu = 6.78$ hours and standard deviation $\sigma = 1.24$ hours.

1. If your sleep time is 8 hours, what percent of students sleep less than you?

The setup for this problem is a classic use of the z-table and properties of a normal distribution. Here, we are given a cut point, x, and need to calculate the z-score associated with it.

(8-6.78)/1.24 = 1.22 → (1.22)/1.24 → 0.98387 = z-score.

Using the table, we can see that 83.65% of observations fall below a z-score of 0.98.

2. Use the "68–95–99.7 rule" to describe the sleep time of college students.

The rule provides a means to identify the approximate cut points in a normal distribution between which 68%, 95%, and 99.7% of observations lie. Identifying the cut points is calculated by adding and subtracting one, two, and three standard deviations from the mean of the distribution, respectively.

68% → x = 6.78 ± 1.24 → 68% of students sleep between 5.54 hrs and 8.02 hrs

95% → x = 6.78 ± (2*1.24) → 95% of students sleep between 4.3 hrs and 9.26 hrs

99% → x = 6.78 ± (3*1.24) → 99.7% of students sleep between 3.06 hrs and 10.5 hrs

3. You take a sample of 100 students and calculate their average sleep time. What is the distribution of all possible sample means of 100 students?

First, we need to calculate the spread associated with samples of 100 students.
Spread = s.d./$\sqrt{n} \rightarrow 1.24/\sqrt{100} \rightarrow 1.24/10 \rightarrow 0.1$ 24 is the spread of the sampling distribution associated with samples of 100. Now, we can use our knowledge that 3.5 standard deviations includes nearly all observations in a normal distribution to calculate the range of possible values.

(1) 3.5*0.124 = 0.434
(2) x = 6.78 ± 0.434 → the distribution of all possible sample means of 100 students ranges from 6.346 to 7.124.

4. You take a sample of 100 students and calculate their average sleep time. What is the probability that the average is below 7 hours?

The problem here has a similar approach and solution to the first problem, only this time, we are using the properties of a normally distributed sampling distribution. Here, we are given an average that will fall somewhere on the distribution of all possible averages. We know that our measure of spread of the sampling distribution of samples of 100 is 0.124. We want to calculate the z-score associated with 7 in the sampling distribution, because again we can use the properties of a normal distribution to see what proportion of samples will fall below 7.

z-score = (x-$\bar{x}$)/s.d. → (7-6.78)/0.124 → 0.22/0.124 → 1.77 = z-score, which tells us there is a 96.16% chance that a sample of 100 students will have a mean below 7 hours of sleep per night.

5. Use the "68–95–99.7 rule" to describe the variability of average sleep time of 100 students.

Here, the approach is again analogous to the second problem. Remember, the sampling distribution mirrors the population distribution. If the population distribution is normal, as specified here, the sampling distribution is also. However, the measure of spread we use for the sampling distribution is the standard deviation divided by the square root of the sample size. Start with calculating the spread of the sampling distribution.

(1) $1.24/\sqrt{100} = 0.124$

68% → x = 6.78 ± 0.124 → the mean of 68% of samples will be between 6.656 and 6.904

95% → x = 6.78 ± (2*0.124) → the mean of 95% of samples will be between 6.532 and 7.028

99% → x = 6.78 ± (3*0.124) → the mean of 99% of samples will be between 6.408 and 7.152

6. You take a sample of 100 students and calculate their average sleep time. The average sleep time is 7 hours. What is the 95% confidence interval for the population average sleep time?

For this, we need to calculate the margin of error associated with samples of 100 students from the population. The formula for a margin of error is the z-score associated with the confidence you are aiming for multiplied by the measure of spread of the sampling distribution. To identify the 95% confidence interval, we need the z-score in which 95% of a distribution falls between. In our case, this means there needs to be 2.5% of the sample on either side of the z-score. Using the z-table, this comes to 1.96 (verify this using the z-table in Stata).

Margin of error (m) = Z * (s.d./$\sqrt{n}$) $\rightarrow 1.96*(1.24/\sqrt{100}) \rightarrow 1.96*0.124 = 0.24304$ is the margin of error. The confidence interval is the calculated by adding and subtracting m from the mean. So,

95% confidence interval ranges from m ± 7 → 6.75696 to 7.24304. NOTE: if you used 2 instead of the more precise 1.96, that would be acceptable as well.