Tests of Significance

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Basic Process

Most policy research involves deceptively simple steps:

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2. State hypotheses about the answer to the question.
3. Collect data that can answer the question.
4. Calculate measures to test hypotheses put forward about the relationship of interest.

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    - Pie and bar graphs - Depict the distribution of a categorical variable.
    - Histogram - Depict the distribution of a quantitative variable.
    - Scatterplot - Depict the relationship between two quantitative variables.
    - Two-way table - Joint distribution of two categorical variables.

Reasoning of Significance Tests

- We have seen that the properties of the sampling distribution of $\overline{x}$ help us estimate a range of likely values for population mean μ
  - Centered on μ
  - Normal distribution with a narrower measure of spread than the population
- Example: You are in charge of ensuring safe streets. You randomly sample speeds of drivers on 4 parts of a main avenue.
- The average speed in your sample was 48 mph. Obviously, we cannot expect every section of the avenue to have the same travel speeds. Thus,
  - Is the somewhat higher speed in your sample due to chance variation?
  - Is it evidence that the city should consider more aggressive enforcement or changes to the streetscape?

Stating Hypotheses

A test of statistical significance tests a specific hypothesis using sample data to decide on the validity of the hypothesis.

In statistics, a hypothesis is an assumption or a theory about the characteristics of one of more variables in one or more populations.

Example: What you want to know: Does the street need more attention for safety reasons?

That same question reframed statistically: Is the population mean μ for the distribution of speeds traveled on the road equal to 35 mph (i.e., the speed limit)?

Stating Hypotheses

The null hypothesis is a very specific statement about a parameter of the population(s). It is labeled H_0.

The alternative hypothesis is a more general statement about a parameter of the population(s) that is exclusive of the null. It is labeled H_a.

Example: Travel speeds on main avenue:

H_0: $\mu = 35mph$ (μ is the average speed of travelers on the road) H_a: $\mu \neq 35mph$ (μ is either larger or smaller)

One-sided and Two-sided Tests

- A two-tail or two-sided test of the population mean has these null and alternative hypotheses:
  - $H_0 : \mu =$ [a specific number] $H_a : \mu \neq$ [a specific number]
- A one-tail or one-sided test of a population mean has these null and alternative hypotheses:
  - $H_0 : \mu =$ [a specific number] $H_a : \mu <$ [a specific number]
  - $H_0 : \mu =$ [a specific number] $H_a : \mu >$ [a specific number]

The FDA tests whether a generic drug has an absorption extent similar to the known absorption extent of the brand-name drug it is copying. Higher or lower absorption would both be problematic, thus we test:
$H_0 : \mu_{generic} = \mu_{brand}$ $H_a : \mu_{generic} \neq \mu_{brand}$ two-sided

How to Choose?

What determines the choice of a one-sided versus a two-sided test is what we know about the problem before we perform a test of statistical significance.
Example: A health advocacy group tests whether the mean nicotine content of a brand of cigarettes is greater than the advertised value of 1.4 mg.
Here, the health advocacy group suspects that cigarette manufacturers sell cigarettes with a nicotine content higher than what they advertise in order to better addict consumers to their products and maintain revenues. Thus, this is a one-sided test: $H_0 : \mu = 1.4\text{mg}$ $H_a : \mu > 1.4\text{mg}$
It is important to make that choice before performing the test or else you could make a choice of "convenience" or fall into circular logic.
In practice, **we want to exercise caution** - a two-sided t-test will thus be preferred in most instances.

The P-Value

The speed of drivers in your city has a known standard deviation of 10 mph.
H_0: $\mu = 35 mph$ versus H_a: $\mu \neq 35 mph$
Tests of statistical significance quantify the chance of obtaining a particular random sample result if the null hypothesis were true. This quantity is the **P-value**.
This is a way of assessing the "believability" of the null hypothesis, given the evidence provided by a random sample.

Interpreting The P-Value

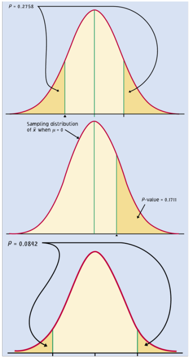
With a small p-value we reject H_0. The true property of the population is significantly different from what was stated in H_0.
Thus, small P-values are strong evidence AGAINST H_0
But how small is small...?

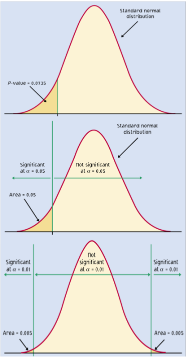
Interpreting The P-Value

P = 0.2758

P = 0.1711

P = 0.0892





P = 0.0735

P = 0.05

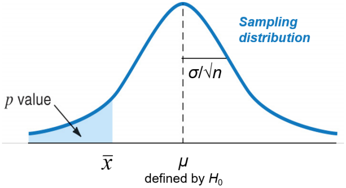
P = 0.01

When the shaded area becomes very small, the probability of drawing such a sample at random gets very slim. Oftentimes, a P-value of 0.05 or less is considered significant: The phenomenon observed is unlikely to be entirely due to chance event from the random sampling.

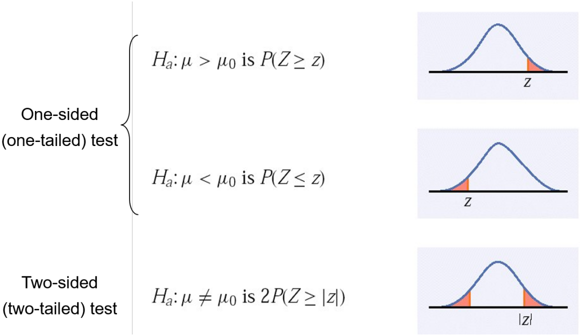
Tests for a Population Mean

The p-value is the area under the sampling distribution for values at least as extreme, in the direction of H_a, as that of our random sample.
Again, we first calculate a z-value and then use a z-table:

$$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$



P-value in one-sided and two-sided tests



To calculate the P-value for a two-sided test, use the symmetry of the normal curve. Find the P-value for a one-sided test and double it.

Does the street need attention for speeding?

- H_0: $\mu = 35mph$ versus H_a: $\mu \neq 35mph$
- What is the probability of drawing a random sample such as yours if H_0 is true?
  $\overline{x} = 48mph$ $\sigma = 10mph$ $n = 4$
  $$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow \frac{48 - 35}{\frac{10}{\sqrt{4}}} \rightarrow 2.4$$
  From a z-table, the area under the standard normal curve to the left of z is 0.9918.
  To the right, this would be 1 - 0.9918 or 0.0082.
  For a two-sided test, we would multiply by 2 (2×0.0082) for a p-value of 0.0164.
  The probability of getting a random sample average this far above μ is so low that we can safely reject H_0.
  We would conclude that the street does need some safety attention.

Steps for Tests of Significance

1. State the null hypotheses H_0 and the alternative hypothesis H_a.
2. Calculate value of the test statistic.
3. Determine the P-value for the observed data.
4. State a conclusion.

The significance level: α

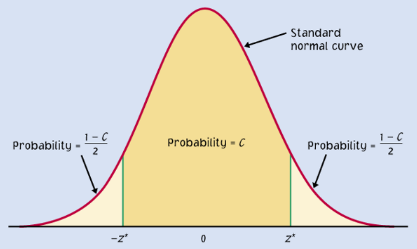
The significance level, α, is the largest P-value tolerated for rejecting a true null hypothesis (how much evidence against H_0 we require). This value is decided arbitrarily before conducting the test.

- If the P-value is equal to or less than α ($P \leq \alpha$), then we reject H_0.
- If the P-value is greater than α ($P > \alpha$), then we fail to reject H_0.

Example: The speed sample p-value was 0.0164. If α had been set to 1%, we would fail to reject the null and the p-value would be insignificant. If α had been set to 5%, we would reject the null and the p-value would be significant.

Confidence intervals and Inference

Because a two-sided test is symmetrical, you can also use a confidence interval to test a two-sided hypothesis. In a two-sided test, C = 1 - α.



Example: $\sigma = 10$ mph: H_0: $\mu = 35mph$ versus H_a: $\mu \neq 35mph$
Sample average 48 mph. 95% CI for μ = 48 mph $\pm$
$1.96 \times \frac{10}{\sqrt{4}} \rightarrow 48mph = \pm 9.8mph$
35 mph is not in the 95% CI (38.2 to 57.8 mph). Thus, we reject H_0.